1. Diffusivity Module Overview

The diffusivity module is responsible for calculating the diffusion coefficients of momentum and temperature for all level interfaces interior to the atmosphere; that is, all model half levels except the top and bottom of the model. Note that the diffusion coefficient for tracers is assumed to be the same as that of temperature.

The diffusion coefficients are set according to their position in the atmosphere: within the planetary boundary layer (PBL), the interface between the PBL and the atmosphere above, and above the PBL. The following sections describe these components.

2. Diffusion in the planetary boundary layer: the K-profile scheme

Within the PBL, there are two methods for calculating diffusivities, a K-profile scheme and the Old Climate model's version. The choice of method is controlled by the namelist parameter "pbl_supersource", which with the default value of FALSE indicates that the K-profile scheme is the default selection. In this section the K-profile scheme is described whereas the old climate-model's version is described in the following section.

Within the boundary layer, the diffusivity of momentum and heat, K_m and K_h respectively, is calculated as a function of height in the PBL, and the surface similarity theory set in the Monin-Obukhov module. If the height of above the surface, z, is less than the PBL depth, h, and is less than the height of the surface layer, $h_{inner} \equiv f \cdot h$, where f is a namelist parameter (called "frac_inner") with a default value of 0.1, then

$$K_m = k u_* z / \Phi_m \quad \text{for} \quad z < h_{inner}, \tag{1}$$

$$K_h = ku_* z / \Phi_h \quad \text{for} \quad z < h_{inner}, \tag{2}$$

where k is the von Karman constant, u_* is the surface friction velocity, and $\Phi_{m,h}$ are the similarity functions for momentum and heat where are evaluated at the height z. At heights above the surface layer but beneath the PBL depth, the diffusion coefficients are set following Troen and Mahrt (1986):

$$K_m = (ku_* z / \Phi_m) \times \left(1 - \left[\frac{z - h_{inner}}{h - h_{inner}}\right]\right)^2 \quad \text{for} \quad h_{inner} \le z < h, \tag{3}$$

$$K_h = (ku_* z / \Phi_h) \times \left(1 - \left[\frac{z - h_{inner}}{h - h_{inner}}\right]\right)^2 \quad \text{for} \quad h_{inner} \le z < h. \tag{4}$$

In (3) and (4), the stability functions are evaluated at a height of h_{inner}.

With this formulation, a crucial aspect is the determination of the PBL depth h. If the namelist parameter "fixed_depth" is set to TRUE, then h is set always to the value of the namelist variable "depth_0". If h is not fixed (the default), then h is determined by a method which is dependent on the stability of the surface layer. If the surface layer is stable or neutral, that is $b_* \le 0$. where b_* is the surface buoyancy scale, then h is set as the height for which a Richardson number, Ri_A, exceeds a critical Richardson number, Ri_h, called "rich_crit_pbl", which has a default value of 1. Ri_A is evalualted as:

$$Ri_A = \frac{g(s_v(z) - s_v(z_s))/(s_v(z_s))}{(U^2(z) + V^2(z))}, \tag{5}$$

where g is the acceleration due to gravity, $s_v \equiv c_p T(1 + 0.608 q_v) + gz$, is the virtual dry static energy, U and V are the zonal and meridional wind speeds, and z_s is set to the (midpoint) height of the lowest atmospheric model level. Some comments regarding the formulation of this Richardson number are in order. In GCMs, it has been found that in very stable conditions traditional Monin-

Obukhov theory does not give enough transfer away from the surface as seems required to prevent the surface temperature from becoming unrealistically cold. As a consequence, it would seem that in highly stable conditions some degree of empiricism is required. The formulation of Ri_A in (5) is designed to minimize the value of the Richardson, and hence maximize the PBL depth in stable conditions. Because this approach is empirical, z_s was not set to 10 meters as in Holtslag and Boville (1993).

For unstable conditions, $b_* > 0.$, the height of the convective boundary layer is determined by find the level of neutral buoyancy of a parcel of air in the surface layer (Beljaars and Viterbo 1999). The virtual dry static energy of this parcel, $s_{v,p}$, is set equal to that of the lowest atmospheric model level plus some excess energy which is depends on the surface buoyancy flux:

$$s_{v,p} = s_v(z_s) + \Delta s_v, \tag{6}$$

$$\text{where} \quad \Delta s_v = \frac{D \cdot \overline{w's_v'}\big|_{z=0}}{w_s} = \frac{D u_* b_* s_v(z_s)}{w_s g}. \tag{7}$$

D is a namelist parameter called "parcel_buoy" which is set to 2 (Beljaars and Viterbo 1999) and z_s is the (midpoint) height of the lowest atmospheric model level. w_s is a velocity scale which is:

$$w_s = u_*/\Phi_m. \tag{8}$$

In (8), the stability function is evaluated at a height of $f \cdot z_{nom}$, where z_{nom} is a nominal boundary layer depth (namelist parameter "znom") whose default value is 1000 m. With $f = 0.1$ by defaul and the default stability function Φ_m of $(1-16\zeta)^{1/4}$, (8) reduces to:

$$w_s = (u_*^4 + 0.64 u_* w_*^3)^{1/4}, \tag{9}$$

$$\text{where} \quad w_*^3 = u_* b_* z_{nom}. \tag{10}$$

However, the module evaluates (8) to allow for varying stability functions.

3. Diffusion in the planetary boundary layer: Old Climate Model Version

As an option, the module may return the old climate model's version of PBL diffusivities. In this formulation the vertical diffusivities depend only on the vertical wind shear:

$$K_m = k \cdot l^2 \cdot \left| \frac{\partial}{\partial z} \vec{V} \right| , \tag{11}$$

and $K_h = K_m$. The mixing length l is specified as:

$$l = kz \quad \text{for} \quad z < h_{inner}\text{, and} \tag{12}$$

$$l = kh(h - z) / (h - h_{inner}) \quad \text{for} \quad h_{inner} \le z < h\,. \tag{13}$$

Although the module permits the use of this formulation with the PBL depth h to be calculated according the method described in the section 3, in the old climate model this formulation has been only been used with a fixed value of h of 5000 m and h_{inner} of 75 m. To recover this option, the namelist parameters "pbl_supersource", "fixed_depth", "depth_0", and "frac_inner" must have the values TRUE, TRUE, 5000., 0.015, repsectively.

4. Entrainment at the top of the planetary boundary layer

In a convectively mixed boundary layer, large-eddies typically overshoot the PBL top and entrain free atmospheric air into the boundary layer. To represent this process, the amount of diffusion at the half-level which nearest the PBL top is set such that the negative buoyancy flux due to entrainment is a specified fraction of the surface buoyancy flux:

$$K_{m,h}\frac{\partial s_v}{\partial z}\bigg|_{z=h} = C_e \cdot \overline{w's_v'}\Big|_{z=0}, \qquad (14)$$

where C_e is a namelist parameter called "entr_ratio" which is specified to be 0.2 (Stull 1988). In terms of model variables,

$$K_{m,h} = \frac{C_e u_* b_* \cdot \Delta z}{g \cdot \Delta s_v / s_v(h)}, \qquad (15)$$

where the Δ is the difference between the full model levels which contain the PBL top. Note that if the surface layer is neutral or stable, $b_* \le 0.$, then $K_{m,h} = 0$.

Note that entrainment is not calculated if the PBL depth is given a fixed value ("fixed_depth" = TRUE). This prevents the calculation of entrainment in the case that PBL diffusivities are calculated according to the old climate model's method.

5. Diffusion in the free atmosphere

Above the PBL, vertical diffusivities may everywhere and always be set to zero by keeping the setting of the namelist parameter "free_atm_diff" at its default value of FALSE. However if it is set to true is calculated according the following method.

Free atmospheric diffusion is specified according to a mixing length formulation:

$$K_m = l^2\left|\frac{\partial}{\partial z}\vec{V}\right| \cdot F(Ri_B) \cdot G(\Delta z), \qquad (16)$$

where l is a mixing length with default value of 30 m (namelist parameter "mix_len"). The Richardson number, Ri_B, is evaluated as:

$$Ri_B = \frac{g\Delta s_v / s_v}{(\Delta U^2 + \Delta V^2)}. \qquad (17)$$

The Richardson number function is given by:

$$F(Ri_B) = \left(1 - \frac{Ri_B}{Ri_f}\right)^2, \qquad (18)$$

where Ri_f is the critical Richardson number beneath which free atmospheric diffusion occurs (namelist parameter “rich_crit_diff” withdefault value of 0.25).

This diffusion is implemented in two ways. In the first method, $G(\Delta z)$ is set to 1, and $K_h = K_m / Pr$, where Pr is a namelist specified Prandtl number (called “rich_prandtl”) with default value of 1. In the second method (selected by setting “free_atm_skyhi_diff” to TRUE), $G(\Delta z) = 1. + 10^{-4}(\Delta z)^{1.5}$, where Δz is the vertical spacing of model levels in km. In addition, the Richardson number used is $Ri_B / G(\Delta z)$, and

$$K_h = K_m \cdot (0.1 + 0.9 \cdot F(Ri_B / G(\Delta z))). \qquad (19)$$

The motivation for this second method can be found in Levy et al. (1982).

6. Background Diffusion

In addition to the above specification of diffusivities, a background level of atmospheric diffusivities (namelist parameters “background_m” and “background_t” with default values of 0.) is specified such that everywhere K_h ≥ background_t and K_m ≥ background_m.

7. References

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